Additional telescope techniques for Laser Guide Star tip–tilt retrieval

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ABSTRACT

Laser Guide Star tip–tilt recovery is a central and crucial problem in the framework of adaptive optics systems capable of full-sky diffraction limited performances. In the last years a number of techniques involving auxiliary telescopes shooting additional Laser Guide Stars or probing portions of the main Laser Guide Star generated by the main observatory has been outlined and published. Two of these techniques, relying on Natural Guide Stars well away from the isokinetic patch of the observed scientific target, use some additional telescopes in the neighborhood to be used as differential tilt telescopes or as auxiliary laser projectors. A comparison of the ground impact and of the maximum achievable performances due to conical anisokinetism problems inherent to the additional telescopes is given.

1. INTRODUCTION

Full sky coverage diffraction–limited capabilities in the visible wavelengths for 8m class telescopes could becomes a reality in the next decade. This scenario could change our vision of the Universe but, however, still requires a lot of engineering development in the field of adaptive optics and Laser Guide Star. In this framework a key–role is played by both conical anisoplanatism and absolute tilt retrieval.1

While there is a substantial agreement over the community that conical anisoplanatism can be solved, at least in principle, by some configuration of multiple LGSs2, there is no clear evidence of the procedure to follow to retrieve a reliable tip–tilt signal from an LGS–based adaptive optics system.

Multicolour LGS, devised in 1992 by Foy and co–workers3, is the more studied technique and it is easy to identify in the high power requirement its major drawback.

In the last three years more than 10 other techniques has been identified and published in the available literature4,5,6. Some of these requires additional telescopes or laser projectors moving around the main observatory. Others requires the adoption of a Natural Guide Stars even if well away from the isoplanatic patch of the scientific target. Two of these techniques, namely the NGS–based perspective ones, requires both the described conditions7,8.
In the following we focus our attention to these perspective technique pointing out their problems and limitations both from the technical and fundamental point of view. By technical limitation we mean limitations due to the available hardware technology. The achievable laser power is one typical example of a technical limitation. By fundamental limitation we mean limitations due to both statistical behaviour of turbulence and finite height of LGS. The conical anisokinetism error is a typical example of fundamental limitations in the case of a single LGS wavefront reconstruction scheme.

2. TECHNICAL ASPECTS

Although the NGS—perspective based techniques using auxiliary telescopes and projectors are essentially the dual of each other, there is a more subtle and substantial difference between the two. In the following, for sake of simplicitness, we'll mention simply the aux telescope or the aux projector technique to refer to these two techniques.

In the aux telescope, in fact, the mesosphere Sodium layer thickness, usually denoted by \( \tau \), doesn't play a significant role in the effectiveness of the technique. The elongation of the LGS as seen by one of the auxiliary telescopes is not useful at all. By the way such elongation has a few side effects:

- The light is spreaded over an angular extension that can be much larger than the isokinetic patch of the aux telescope and this translates into a smaller usable amount of resonance backscattered photons;
- The tilt can be estimated only in one direction, so that two separate aux telescopes are needed to sample the bidimensional tilt;
- Finally, the LGS elongation can be useful to limit the track extension and movement of the aux telescope. Any component of the movement projected along the LGS strip apparent elongation can be easily absorbed by the fact that the NGS is sensed in the background of a different portion of the LGS strip.

The situation is totally different for the aux projector case. In the latter, in fact, the strip appearance due to the thickness extension of the mesospheric Sodium layer plays a crucial role. The two cases are summarized in Fig.1a and Fig.1b where such substantial difference is specifically pointed out. In other words one can say that in the limit case \( \tau \to 0 \) the aux telescope technique is still working (and, from a certain point of view, in a better way: for instance only one aux telescope instead than two are required) while the aux projector doesn't produce any useful tip—tilt signal (when \( \tau \to 0 \) the distance \( s \) between the main telescope and the projector doesn't care for the geometry of what is apparently seen from the main observer).

It is useful to introduce a parameter \( \gamma \) defined as the ratio between the thickness \( \tau \) and the mean altitude \( H \) of the Sodium layer:

\[
\gamma = \frac{\tau}{H}
\]  

(1)

Because the Sodium extension is smoothly changing with altitude, such a parameter will depends upon the adopted criterion to define \( \tau \). The latter depends, for instance, upon the brightness threshold adopted for the strip elements. Moreover, it is known that \( \tau \) is a seasonally variable parameter. Suitable values lies in the range \( \gamma = 0.1\ldots0.2 \).

In order to compare the two techniques from the point of view of the ground occupation one has to relate the ground elongations \( l \) and \( s \) with respect to the number of expected useful NGSs that one can find out. In both
cases it is essential to figure out the proportionality between the aperture diameter $D$ or $d$ and the number $n$ of useful NGSs in the same sky patch. Assuming an isotropically filled space of stars with the same luminosity function $n D^3$ relationship is easily obtained. However light extinction due to the interstellar dust and the finite extension of the milky way provides a cut–off to eq.(2). We’ll write this fact into the form:

$$ n \propto D^3 $$

(2)

where $\mu = 0$ means no extinction and deep of observations smaller than the deep of the Galaxy in the observed position. The parameter $\mu$ will depends upon the magnitude range under examination and upon the direction in the sky where the observation is currently made. We can assume reasonable values of $\mu = 0 \ldots 0.5$. With all these assumptions we can easily work out for the aux telescopes technique the max off–axis angle $\theta$ with respect to the observed target as:

$$ \theta \approx \frac{l}{H} $$

(4)

and finally:
\[ n_{\text{tel}} \approx \theta^2 d^{3-\mu} = \frac{l^2 d^{3-\mu}}{H^2} \] (5)

where we omitted any common proportionality constant like \( \pi \) or the one inherent in eq. (3). For the aux projector case the situation is slightly more complicated. In order to compute the max off-axis angle \( \psi \) we introduce the apparent extension \( \eta \) where:

\[ \eta \approx \gamma s \] (6)

and

\[ \psi \approx \frac{\eta}{H} \approx \frac{\gamma s}{H} = \frac{\tau s}{H^2} \] (7)

Finally:

\[ n_{\text{proj}} \approx \psi^2 D^{3-\mu} = \frac{\gamma^2 s^2 D^{3-\mu}}{H^2} \] (8)

Equating \( n_{\text{tel}} = n_{\text{proj}} \) one obtains the following relationship:

\[ \frac{l}{s} = \gamma \left( \frac{D}{d} \right)^{\frac{3-\mu}{2}} \] (9)

The last equation leads to a number of useful considerations. For instance it sets the gain in ground coverage between the two techniques. A reasonable choice of \( D/d = 10 \), in fact, leads to \( l/s \approx 6.6 \) under the most optimistic case \( \gamma = 0.2 \) and \( \mu = 0 \). It is to be recalled that the ground occupation ratio scales with \( (l/s)^2 \) in order to have an idea of the cost-effectiveness achievable with one technique rather than the other.

From eq. (9) one can also establish which is the aux telescope diameter \( d \) that makes the aux telescope technique less demanding from the same point of view than the aux projector one. Such diameter is given imposing \( l = s \) and gives:

\[ d = D\gamma^{\frac{2}{3-\mu}} \] (10)

There are, however, other technical constraints that one has to keep in mind. In the aux projector technique the diameter of the projector has very little influence, if nothing at all, on the performance of the tip-tilt retrieving scheme. Moreover it doesn’t require any high-speed information transmission between the movable units and the main observer such as required by the aux telescope scheme.

Both techniques could adopt a single aux unit to retrieve tip-tilt in the two directions, provided a pulsed laser and a gating system are introduced in the tip-tilt retrieval scheme. Rayleigh disturbances in the scientific field of view is the same, while laser power requirement, Rayleigh pollution in the observatory neighborhoods, satellite and aviation safety problems are thriced in the aux projector technique with respect to the aux telescopes one.
3. FUNDAMENTAL LIMITATIONS

The fundamental limitation in retrieving the NGS tilt using tilt measurement done on a single LGS is due to the finite height of the laser spot. In fact, even assuming that the laser spot position on the scatterer layer is known the plane and spherical wave propagating from the NGS and LGS respectively sample a different atmospheric phase perturbation. This introduce a difference between the downward LGS and the NGS tilt as briefly shown in Fig.2 in the case of a single turbulent layer.

As explained previously these techniques both estimate the NGS tilt in a two-step process. First, the upward tilt of the LGS is estimated using a background star that can be well outside of the isokinetic patch of the telescope considered. Second, this information is used together with the LGS tilt as seen on the main telescope to obtain the LGS downward tilt that represent our estimation of the NGS tilt. Each of these step introduce an error in our estimate of the NGS tilt. The second step introduced an error that is equal for the two techniques and is due to the difference between the NGS and LGS downward tilts as seen from the main telescope. As reported in Ref.9 the tilt phase error variance for a sodium guide star can be expressed as

\[ \sigma_f^2 = 0.59 \left( \frac{D}{d_0} \right)^{5/3} \]  

where D is the telescope diameter and \( d_0 \) is the LGS parameter introduced by Fried\(^{10} \). The error introduced in the first step is, as before, due to focus anisokinetism effect but is different in magnitude for the two techniques. Both techniques evaluate the upward LGS tilt performing a differential tilt measurement between a suitable NGS and a portion of the elongated laser spot. In this differential measurement the NGS tilt and the LGS downward tilt cancel out giving the value of the upward LGS tilt. However this is not completely true because of the focus anisokinetism effect that prevents the NGS and LGS downward tilt to exactly cancel out. The magnitude of this effect depends, as shown before, on the diameter of the telescope performing the measurement. This diameter is the main telescope diameter in the case of the auxiliary projector technique and the auxiliary telescope diameter in the case of the auxiliary telescope technique. It is easy to see that this second error is uncorrelated with the previous one so that we can sum up the variances to obtain the overall phase error for the two techniques that we indicate \( \sigma_{tel}^2 \) and \( \sigma_{proj}^2 \). Using eq.(11) we obtain
Figure 3: The phase error variance contribution for the two techniques described in the text at different additional telescopes diameter. The thinner line is for the auxiliary telescopes while the thicker line is for the auxiliary projectors.

\[ \sigma^2_{proj} = 2 \times 0.59 \left(\frac{D}{d_0}\right)^{5/3} \]  \hspace{1cm} (12)

and

\[ \sigma^2_{aux} = 0.59 \left(\frac{D_{main}}{d_0}\right)^{5/3} \left[1 + \left(\frac{D_{main}}{D_{aux}}\right)^{1/3}\right] \]  \hspace{1cm} (13)

Here a factor, \((D_{main}/D_{aux})^2\) has been introduced to rescale the phase tilt error variance between the auxiliary and the main telescope. Introducing the quantity \(\sigma^2_{fa}\) defined as the phase error variance due to focus anisokinetism on the main telescope, we can rewrite the two formulas so that

\[ \sigma^2_{proj} = 2\sigma^2_{fa} \]  \hspace{1cm} (14)

and

\[ \sigma^2_{tel} = \sigma^2_{fa} \left[1 + \left(\frac{D_{main}}{D_{aux}}\right)^{1/3}\right] \]  \hspace{1cm} (15)

These two last equations show how the NGS tilt estimation obtained using the aux projector technique is more accurate of the one obtained using the aux telescope technique. Fig. 3 report numerical results for \(\sigma^2_{proj}\) and \(\sigma^2_{tel}\) in the case of an Hufnagel-Valley profile\(^1\) normalized to give \(r_0 = 20\) cm at 0.5\(\mu\)m and \(d_0 = 3\) m.
4. CONCLUSIONS

We have compared two NGS-perspective based technique using the same formalism and the same atmospheric conditions. The results presented here is somewhat preliminary and, moreover, still relys on a number of parameters that are to be well-defined for each application case in order to figure out some definitive conclusion. The auxiliary projector approach is somewhat superior in terms of achievable performances and required ground coverage. Such superiority is expressed by relationships and number discussed in the text but, still, these are to be balanced by several technical issues. As a very preliminar conclusions it can be said that the auxiliary telescope technique remains very attractive whenever laser resources are lacking and performances very close to the diffraction limited are not strictly required.

5. REFERENCES